Wyburns Primary

Written Calculations Policy

Our school works in accordance with the PREVENT Duty and approaches this issue in the same way as any other child protection matter. Any concerns that one of our pupils is at risk in this respect, will be referred to Children's Social Care in line with the SET procedures.

Wyburns Primary School is a Rights Respecting School. Our policies are underpinned by the UNCRC.

Article 29(goals of education)

- *Education must develop every child's personality, talents and abilities to the full. It must encourage the child's respect for human rights, as well as respect for their parents, their own and other cultures, and the environment.*

Rationale and Aims

This policy is intended to provide a framework for the teaching of the four rules of number. More specifically, it has been compiled to create a bridge between mental and written methods, whilst addressing the need for clear progression and consistency in teaching approaches. Although there is reference made to expectations for particular year groups, it is recognised that children do not progress at the same rate. This policy, therefore, attempts to provide guidance for the progression of the majority of pupils.

The document draws directly from the Primary Framework Guidance Paper on Calculation and the NGFL suggested policy, building upon the approaches suggested therein. It acknowledges that pupils' mental methods and understanding are key factors in their success at written methods for calculation (the mental skills required to teach each operation are outlined in the appendix). Furthermore, it highlights the importance of the relationships between addition, subtraction, multiplication and division and why these should be taught in conjunction. Pupils should know about the inverse relationships between addition and subtraction, and multiplication and division. They should also be encouraged to understand that multiplication is repeated addition and that division is repeated subtraction.

Finally, by the end of KS2 the majority of children should be able to make an informed choice as to what method of calculation (mental, written or calculator) they will use. When choosing written, they will be able to apply an efficient method for the four rules of number with confidence and understanding.

Key Objectives for All Four Operations

The following key objectives pertain to the teaching of written methods for all four operations. The first three are vital in helping the children to develop their knowledge of the number system and to reason about numbers and the way in which we calculate.

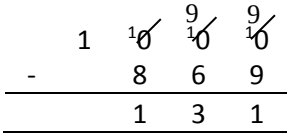
Horizontal Presentation

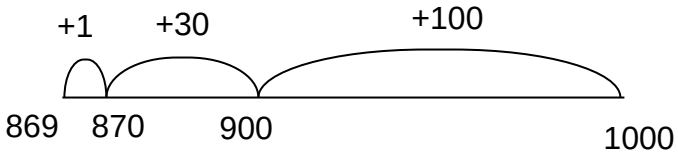
Pupils are presented with written calculations in the horizontal format. This enables them to concentrate on the value of the numbers within the calculation rather than focussing on separate digits. By considering the answer in this format, pupils are better able to see what method of calculation is appropriate and whether or not their answer is reasonable.

Selecting Best Method of Calculation

Pupils are taught that written methods are a tool for calculation and as such are not always necessary. When considering calculations in the horizontal format, pupils must be encouraged to decide whether or not a written method is necessary and give reasons why. Without this focus, pupils will often tackle problems using written methods that are much easier to calculate mentally.

Example: 1000 – 869 =





Estimate, Reason, Reflect

We encourage pupils to go through the process of estimating an answer before tackling a written calculation. Once again, this will better enable children to understand how reasonable their final answers are. This involves three stages: estimating; giving reasons for that estimate; and then reflecting on their final answer in light of the estimate.

Calculators

Pupils are given opportunities to use calculators in every year group. This can be implemented in a number of ways: in role play; using them for checking using the inverse operation; as a tool for tackling worded problems where the focus is not the calculation but the understanding of the problem in context.

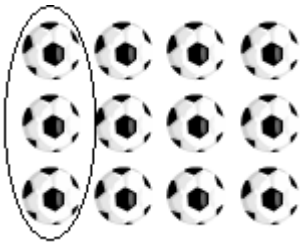
Written Methods for the Four Rules of Number

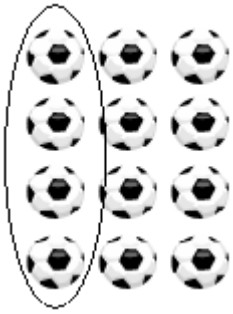
The next section outlines the developmental stages in progression of written methods for each of the four operations. It is important to note at this stage that not all children will progress at the same rate. As such we must assess the pupils' understanding and address our teaching to their needs. There is little value in moving pupils on before they have a sound grasp of the concepts being taught.

For each operation there is a model providing guidance on the stages that the majority of the year group should be working at. Clearly pupils with SEN are likely to require more time and input at the earlier stages, which may be supported by the Wave 3 materials. Conversely more able pupils are likely to be ready to move forward sooner and can therefore progress to the next step. However, it can be beneficial to enrich the learning of these children at a given level through application of their knowledge in a broader range of contexts, rather than to rush forward to the next stage.

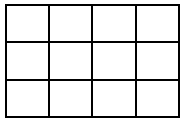
Pupils' understanding of how written methods work will largely depend on how successfully we can relate mental methods, supported by jottings, to the more formal written methods. We need to provide pupils with clear visual images for their mental methods that will enable them to have better conceptual understanding when moving to formal written methods.

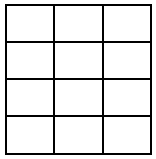
For example, when considering multiplication, the visual image of an array can be very powerful in helping children to understand that a x b = b x a.

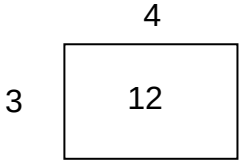
3x4= 

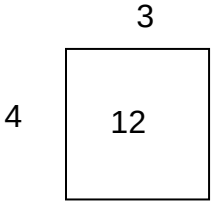
4x3= 

The next step would be to relate this to the area model, firstly with a grid and then moving onto a blank rectangle with the answer in the middle.









This is now beginning to relate very closely to the grid method for multiplication. The ability to draw on visual images when considering a written calculation can support pupils' understanding.

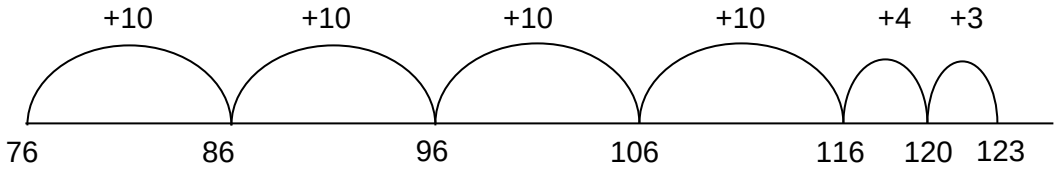
Addition

Step 1

Mental methods need to be secure.
Pupils will be encouraged to start with the *largest* number; this may require them to reorder the calculation.
When calculating, pupils will build on mental methods by counting up in tens first.
Number lines should be used.

Example:

47 + 76 =
76 + 47 =
76 + (10 + 10 + 10 + 10) + 7

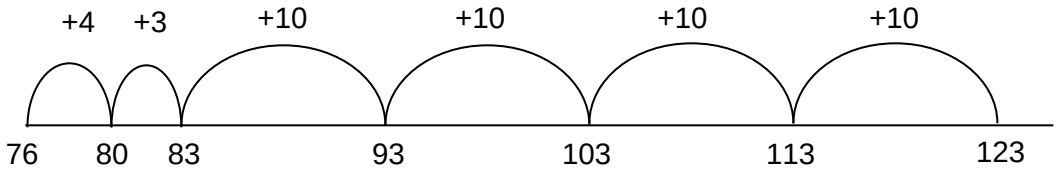


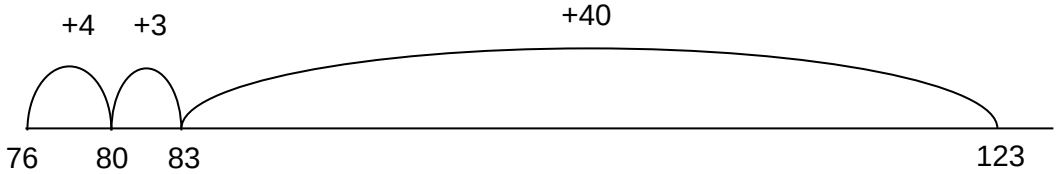
Step 2

Using partitioning with the *least* significant number being added first.
Pupils will be encouraged to add the least significant digit first so that when they make the step to a formal compact layout it is not unusual for them to add the units first. Number lines should also be used, starting with small jumps getting larger as confidence grows.

Example:

47 + 76 =
(76 + 7) + 40



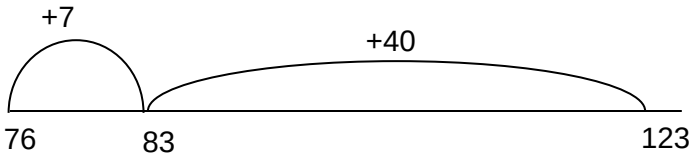


Step 3

Expanding the layout of a calculation, showing the addition of the tens and the units separately, but continuing to start with the least significant digits first. Calculations can be checked by using the empty number line; this allows pupils to check the new method that they are learning by applying a method that they are confident with.

Example:

	7	6
+	4	7
	1	3
1	1	0
1	2	3



Extension of step three: larger numbers and more than two numbers being added at a time.
Pupils may be encouraged to use checking methods such as the inverse operation; again, this should be done with an empty number line.

Step 4

Using the compact layout alongside the previous layout - moving onto the compact layout on its own.

Example:

(1carry)

	4	7
+	2	6
	1	3
	6	0
	7	3

	4	7
+	2	6
	7	3
	1	

(2 carries)

	4	7
+	7	6
	1	3
1	1	0
1	2	3

	4	7
+	7	6
1	2	3
1	1	

Step 5

Extending step four: with larger numbers and decimal numbers. Pupils may need to initially to revert to a more expanded layout before being confident and independent with the compact layout.

Example:

	4	1.	7
+	2	2.	6
		1.	3
		3.	0
	6	0.	0
	6	4.	3

	4	1.	7
+	2	2.	6
	6	4.	3
		1	

NB Money

Calculations involving money will likely require pupils to carry out additions involving decimals before reaching Step 6 of this model. It is advised that these are carried out on the empty number line during Year 3. In Year 4 they can then be introduced using the expanded vertical layout, without any carrying/adjustment. Finally, towards the end of Year 4, carrying/adjustment can be introduced, when the children are more confident with vertical addition.

Subtraction

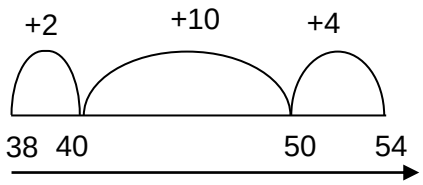
Step 1

Empty Number Line

Build on the mental methods the children already have, including the comparison of addition and subtraction. Use empty number lines or bead strings to count on to the nearest ten, then hundred if it is needed. The ***counting on*** model relates more closely to the mental methods that pupils use. A very few children may need to use a full number line with jumps before the start of step one. With this method, we count on from the smaller number.

Example:

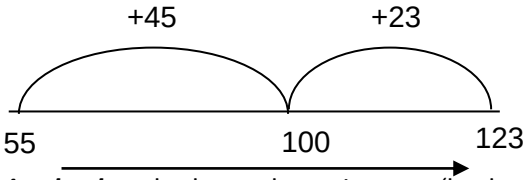
54 – 38 = 16



Progressing to fewer steps; using number facts such as pairs of numbers to 10 and 100, with larger numbers.

Example:

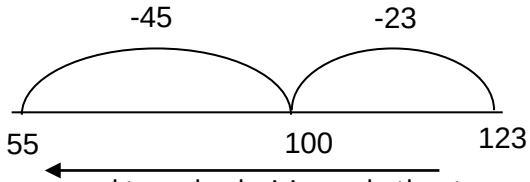
123 – 55 =



Introduce ***counting back*** and relate subtraction to a 'backwards' movement on the empty number line. This method counts back starting with the larger number.

Example:

123 – 55 =

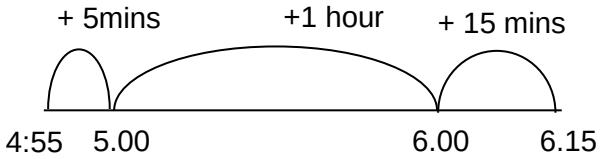


Children will be encouraged to make decisions whether to count on or back depending upon the numbers involved in the calculation.

The empty number line is useful to use at any stage of development for the areas of decimals (money) and particularly for time.

Example:

A programme begins at 4.55pm and finishes at 6.15p.m. How long is it on for?



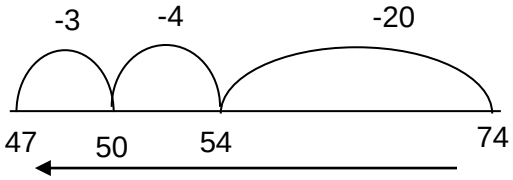
Step 2

Partitioning

Subtraction can be recorded using partitioning to write equivalent calculations that can be carried out mentally.

Example:

74 – 27 = 74 – 20 – 7 = 54 – 7 = 47



This method of recording links to counting back on the number line.

Step 3

Expanded layout

Partitioning the numbers into tens and ones and writing one under the other mirrors the column method, where units are placed under units and tens under tens. This does not link directly to mental methods of counting back or up but parallels the partitioning method for addition. It also relies on secure mental skills. The expanded method leads children to the more compact method so that they understand its structure and efficiency. The amount of time that should be spent teaching and practicing the expanded method will depend on how secure the children are in their recall of number facts and with partitioning.

Example:

74 – 27 =

$$\begin{array}{r} 70+4 \\ -\ 20+7 \\ \hline \end{array}$$

$$\begin{array}{r} {\scriptstyle 60}\quad {\scriptstyle 14} \\ \cancel{70}+\cancel{4} \\ -\ 20+7 \\ \hline 40+7 \end{array}$$

$$\begin{array}{r} {\scriptstyle 6}\ {\scriptstyle 14} \\ \cancel{7}\ \cancel{4} \\ -\ 2\ 7 \\ \hline 4\ 7 \end{array}$$

741 – 367 =

$$\begin{array}{r} 700+40+1 \\ -\ 300+60+7 \\ \hline \end{array}$$

$$\begin{array}{r} {\scriptstyle 600}\quad {\scriptstyle 130}\quad {\scriptstyle 11} \\ \cancel{700}+\cancel{40}+\cancel{1} \\ -\ 300+60+7 \\ \hline 300+70+4 \end{array}$$

$$\begin{array}{r} {\scriptstyle 6}\ {\scriptstyle 13}\ {\scriptstyle 11} \\ \cancel{7}\ \cancel{4}\ \cancel{1} \\ -\ 3\ 6\ 7 \\ \hline 3\ 7\ 4 \end{array}$$

Step 4

Column Method (Decomposition)

Once the children have secure understanding of place value through the expanded method, they should move on to the more efficient decomposition method.

Example:

74 – 27 =

$$\begin{array}{rcc} & \cancel{6}^{5}7 & {}^{1}4 \\ - & 2 & 7 \\ \hline & 4 & 7 \end{array}$$

741 – 367 =

$$\begin{array}{rccc} & \cancel{6}^{5}7 & \cancel{13}^{12}4 & {}^{1}1 \\ - & 3 & 6 & 7 \\ \hline & 3 & 7 & 4 \end{array}$$

Multiplication

Step 1

Partitioning

Building on the mental method of multiplication by partitioning, children should be encouraged to record the stages as jottings.

Example:
38 x 7 =

30 + 8
↓ ↓ x 7
210 + 56 = 266

This method reflects the distributive law and, through the use of brackets, can be developed into a more formal written method.

Step 2

Grid Method

The grid layout allows for an expanded format to be used and links closely to partitioning. Note that the larger number is placed at the top of the grid.

38 x 7

(The addition part could be carried out on empty number line.)

x	30	8	
7	210	56	266

Step 3

The grid layout extended to larger numbers and decimals.

27 x 56=

x	50	6	
20	1000	120	1120
7	350	42	392
			1512

3.5 x 12 =

x	10	2	
3	30	6	36
0.5	5	1	6
			42

This method may be the main method for pupils whose progress is slow; for those who are likely to be achieving a low level 4 by the end of KS2.

Step 4

Expanded Short Multiplication

The next step is to move the number being multiplied (38 in the example shown) to an extra row at the top. Presenting the grid this way helps children to set out the addition of the partial products 210 and 56. Children should be encouraged to begin the multiplication with the least significant digit, as this leads on to the more efficient short multiplication method.

Example:

	3	8
x		7
	5	6
2	1	0
2	6	6

Most children should be secure in this method by the end of Year 4.

Step 5

Short Multiplication

The final stage requires pupils to be able to mentally apply the column method for addition with carrying.

38 x 7 =

	3	8
x		7
2	6	6
	5	

56 x 27 = (approximately 60 x 30 = 1800)

		5	6
	X	2	7
	3	9	2
1	1	2	0
1	5	1	2
	1		

The carrying when totalling the partial products of 392 and 1120 would be done mentally.

Division

Mental Division Using Partitioning

Relating division to the mental methods of partitioning and multiplication is important. To work out TU ÷ U mentally children should be encouraged to use known multiplication facts. In this case, 10 x 7 = 70.

84 ÷ 7 = 84

70 + 14

↓ ↓ ÷ 7

10 + 2 = 12

A further way to make the link to multiplication explicit is to record the division in a grid, which relates very closely to the grid method for multiplication.

x	?	?
7	70	14

→

x	10	2
7	70	14

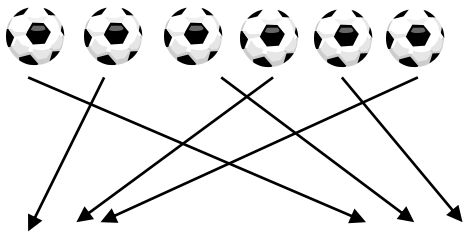
10 + 2 = 12

Step 1

Sharing and Grouping

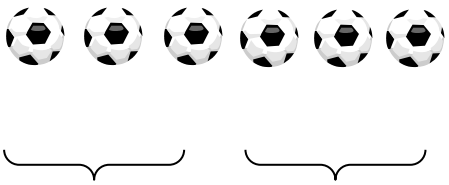
Building on mental methods, children should be clear about the differences between sharing and grouping. This can most simply be taught through the use of clear visual examples.

Sharing ("One for me, one for you")



6 ÷ 2 = 3

Grouping ("How many times can I subtract…from…?")

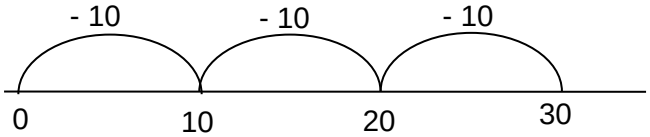


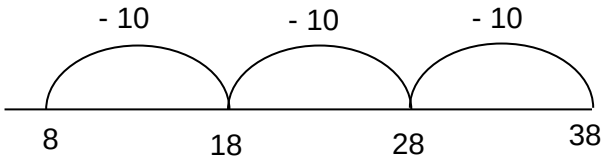
$$3\overline{)6}\quad 2$$

Step 2

Repeated Subtraction

Using bead strings, the empty number line or calculators, children should be taught to understand division as repeated subtraction. This can then be extended to incorporate the idea of remainders.





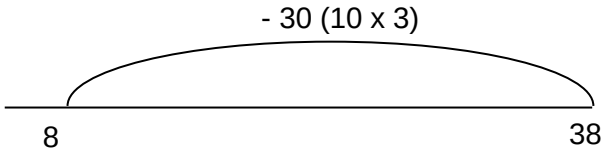
How many times can I take 10 away from 30? 3

$$10\overline{)30}\quad 3$$

How many times can I take 10 away from 38? 3 with 8 remaining

$$10\overline{)38}\quad 3\ r8$$

Larger steps can be introduced at this stage.

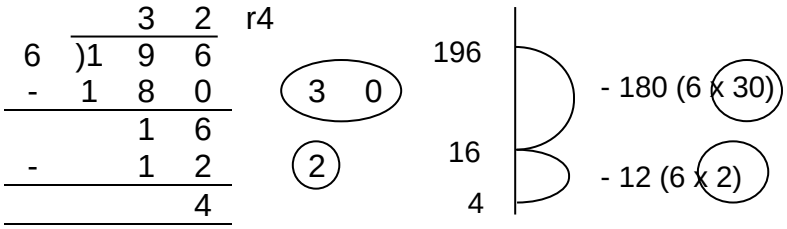


Step 3

Relating Empty Number Line to Chunking Method

Once the children have understood repeated subtraction, the chunking model can be introduced alongside a vertical empty number line. This helps to support children with the recording by relating it to a familiar method.

This should be extended to larger numbers, with the subtraction carried out on an empty number line or calculator.



When the children are secure in their methods for column multiplication, the chunking method should be used on its own, without the support of the empty number line.

Step 4

Short Division

The short division model should be introduced as a more compact and efficient method for division of numbers greater than 100 by a single digit. Although no chunking is involved, the model can still be related to repeated subtraction by considering how many times the U can be subtracted from each partition.

```
      1   3   0   r2
  6 )7  18  2
```

Step 5

Long Division

The final stage moves on to division by a number greater than 10 and this requires the long division model, which closely relates to chunking.

```
          2   3   r8
  2   4 )5  6   0
      - 4   8   0
    -------------
          8   0
      -   7   2
    -------------
              8
    -------------
```

APPENDIX

Source: DfES (2006) Primary Framework for Literacy and Mathematics: Guidance on Calculation

Mental methods of calculation

Oral and mental work in mathematics is essential, particularly so in calculation. Early practical, oral and mental work must lay the foundations by providing children with a good understanding of how the four operations build on efficient counting strategies and a secure knowledge of place value and number facts. Later work must ensure that children recognise how the operations relate to one another and how the rules and laws of arithmetic are to be used and applied. Ongoing oral and mental work provides practice and consolidation of these ideas. It must give children the opportunity to apply what they have learned to particular cases, exemplifying how the rules and laws work, and to general cases where children make decisions and choices for themselves.

The ability to calculate mentally forms the basis of all methods of calculation and has to be maintained and refined. A good knowledge of numbers or a 'feel' for numbers is the product of structured practice and repetition. It requires an understanding of number patterns and relationships developed through directed enquiry, use of models and images and the application of acquired number knowledge and skills. Secure mental calculation requires the ability to:

- recall key number facts instantly - for example, all addition and subtraction facts for each number to at least 10 , sums and differences of multiples of 10 (Year 3) and multiplication facts up to 10 × 10:
- use taught strategies to work out the calculation - for example, recognise that addition can be done in any order and use this to add mentally a one-digit number or a multiple of 10 to a one-digit or two-digit number, partition two-digit numbers in different ways including into multiples of ten and one and add the tens and ones separately and then recombine, when applying mental methods in special cases;
- understand how the rules and laws of arithmetic are used and applied - for example, to add or subtract mentally combinations of one-digit and two-digit numbers, and to calculate mentally with whole numbers and decimals.