PROGRESSION FROM MENTAL TO WRITTEN METHODS OF ADDITION AND SUBTRACTION

BEFORE PROGRESSING THROUGH THE STAGES OF WRITTEN CALCULATION, THE FOLLOWING MENTAL SKILLS ARE CRUCIAL PREREQUISITES:

Remember: **Best Friend in Maths is 10 and 2nd best friend is halving and doubling (h/d).**

Learning

- It is crucial to know or be able to derive key number facts –
  Year 1 +/- totals to 10 (instant recall) then +/- facts within 10

  Year 2 +/- within at least 10 (instant recall) and totals to 20 (instant recall)

  Year 3 +/- within 20 (instant recall or rapid mental calculation using known facts and place value)

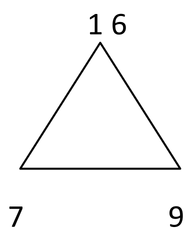
  Year 4 onwards 2-digit +/- 2-digit number (mental calculation using known facts and place value, possibly supported by a jotting such as a number line NOT a vertical method)

  Year 5 and 6 Consolidation and practise of the previous key facts

- Place value and partitioning MUST be clearly understood and explained using the appropriate mathematical vocabulary.

Teaching

- The number line must be modelled as an image to support mental subtraction from Year 1 to Year 6.
- Jottings, including the use of arrows prior to formal written methods, must be modelled as a clear image/strategy for mental calculation.
- Teach the ‘three related numbers’ so that links between the two operations are recognised, e.g.



7 + 9 = 16
9 + 7 = 16
16 – 9 = 7
16 – 7 = 9

Always present calculations horizontally in order to consider mental calculations first.

Always think:

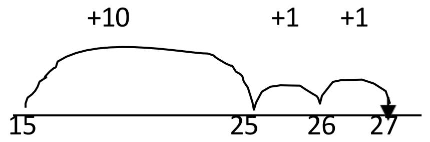
1. *Can I do it mentally?*
2. *Can I do it with a jotting?*
3. *Do I need a written method?*

ADDITION

1. COUNTING ON IN 1s

2. COUNTING ON IN 10s AND 1s

15 + 12 = 27



Year 1

Model with a bead bar, using a 'tens-catcher' to model counting on in 10s)

Number line Teaching Points:

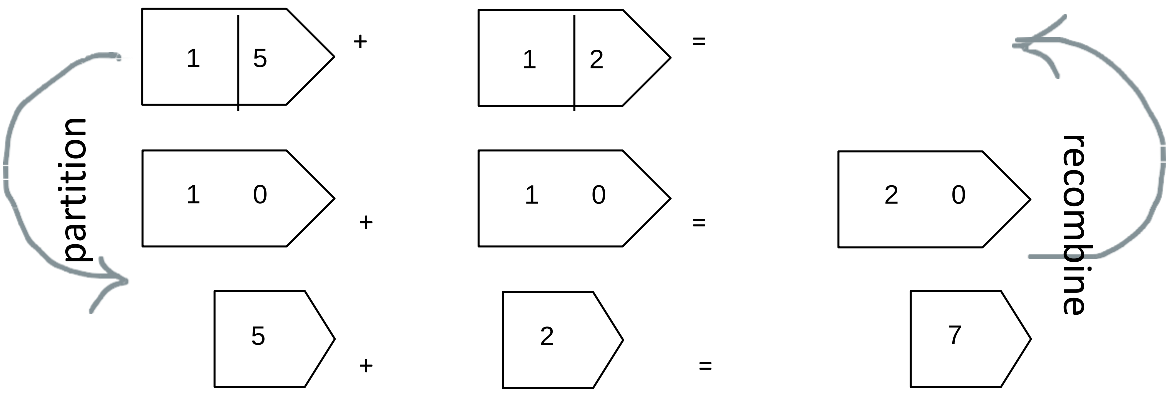
- Always work with numbers reading from left to right (smallest to largest), whatever the operation of the calculation.
- Numbers ('landmarks') are written below the line.
- MUST show an arrow at the end of the 'jumps' to show direction.
- Size of the 'jumps' are written above the 'jumps'.

AS SOON AS CHILDREN ARE COMPLETELY SECURE IN THEIR UNDERSTANDING OF PLACE VALUE AND THEY USE THE APPROPRIATE VOCABULARY TO RECOGNISE AND EXPLAIN 24 IS "2 TENS AND 4 ONES" THEN MOVE ON TO CALCULATING BY:

3. PARTITIONING AND RECOMBINING

Year 2

15 + 12 = 27



Model and practise with place value arrow cards, using known facts and place value to calculate each step.

Moving onto recording as:

34	+	23	=	57
30	+	20	=	50
4	+	3	=	7

It is important to model the use of the arrow combination in calculation jottings. These can later be applied to other jottings and concepts.

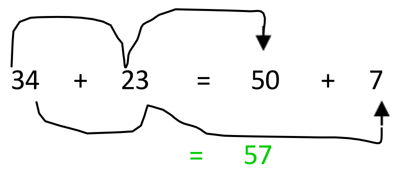
Express this as:
"30 add 20 equals 50"
AND as:
"3 tens add 2 tens equals 5 tens or 50"

Write equals under equals rather than write across the page. This organised layout makes the question and final answer clear.

As well as place value arrow cards, also model this on a bead bar and practise on 100-beadstrings, showing the 'collection' of 10s and then the ones. i.e "3 tens and 2 tens makes 5 tens, which is 50. Then 4 and 3 makes 7 ones. Altogether we can see 5 tens and 7 ones, which is 57." Check by counting in tens and ones along the 100 bead bar.

Progression in Calculations

Moving onto a more efficient recording:



"chains"

Year 2 TAF: Add any 2 two-digit numbers using an *efficient strategy*, explaining their method verbally, in pictures or using apparatus (e.g. 38 + 46).

Years 2 & 3 onwards and mentally through KS2 for addition of 2-digit numbers

Stop using the 'chain' as soon as the concept of 'chaining up the tens and then the ones' is grasped. The chain acts as a reminder model and is not needed on the jotting once the child is secure with the strategy. Eventually, the jotting won't be needed at all as the calculation will be done entirely mentally (expectation by Year 4).

PROGRESSIVE STEPS FOR EACH STAGE OF PARTITIONING AND RECOMBINING:

- Not crossing tens, e.g. 24 + 35
- Crossing tens, e.g. 24 + 17
- Crossing hundreds only, e.g. 56 + 61
- Crossing tens and hundreds, e.g. 76 + 85

CONTINUE USING THIS NOTATION/JOTTING (WITH/WITHOUT CHAIN ACCORDING TO NEED) WHEN EXTENDING TO MENTAL ADDITION OF:

- pairs of 3-digit numbers, if appropriate (Year 3+)
- pairs of decimals when decimal place value knowledge is secure (Year 5+)

When crossing ones, express this as:
"3 tenths add 8 tenths equals 11 tenths, which is 1 whole one and 1 tenth, which is 1.1"

ONLY MOVE TO THE NEXT STAGE IF THE NUMBER FACTS WITHIN 20 ARE KNOWN AND TU + TU CAN BE CARRIED OUT MENTALLY (possibly supported by a jotting)

If the calculation should be possible mentally then do not give it to practise vertical calculation, e.g. TU + TU should not be calculated vertically. Consider use of numbers carefully, including the progressive steps within a stage. When secure, teach children to choose the most appropriate method and include calculations that can be carried out mentally/with a jotting so that children 'spot' them.

Remember: ALWAYS PRESENT CALCULATIONS HORIZONTALLY IN ORDER TO CONSIDER MENTAL CALCULATIONS FIRST.

Year 3

4. ADDING THE MOST SIGNIFICANT DIGITS FIRST

123 + 145 = 268

```
  1 2 3
+ 1 4 5
  2 0 0
    6 0
      8
  2 6 8
```

Working from left to right:
"1 hundred + 1 hundred is 200
2 tens + 4 tens = 6 tens, which is 60 or 20 + 40 is 60
3 + 5 is 8"

'Read' the answer from left to right, using knowledge of place value:
"two hundred and sixty-eight" NOT adding up columns for the final answer.

PROGRESSIVE STEPS FOR EACH STAGE OF ADDING THE MOST SIGNIFICANT DIGITS FIRST:

- Not crossing tens e.g. 132 + 146
- Crossing tens e.g. 127 + 154
- Crossing hundreds only e.g. 178 + 131
- Crossing tens and hundreds e.g. 175 + 147

If a calculation should be possible mentally then do not give it to practise vertical calculation, e.g. TU + TU should not be calculated vertically. Consider use of numbers carefully, including the differentiation steps within a stage. When secure, teach children to choose the most appropriate method and include calculations that can be carried out mentally/with a jotting so that children 'spot' them.

5. ADDING THE LEAST SIGNIFICANT DIGITS FIRST

Year 3

323 + 245 = 568

```
  3 2 3
+ 2 4 5
      8
    6 0
  5 0 0
  5 6 8
```

Working from right to left:
"3 + 5 is 8
2 tens + 4 tens = 6 tens, which is 60 or 20 + 40 is 60
3 hundred + 2 hundred is 500"

'Read' the answer from left to right in an upwards direction, using knowledge of place value:
"five hundred and sixty-eight" (still NOT adding up columns for the final answer)

PROGRESSIVE STEPS FOR EACH STAGE OF ADDING THE LEAST SIGNIFICANT DIGITS FIRST:

- Not crossing tens — e.g. 432 + 246 Consider mental strategy of partitioning and recombining first
- Crossing tens — e.g. 527 + 354
- Crossing hundreds only — e.g. 378 + 431
- Crossing tens and hundreds — e.g. 875 + 247

ENSURE THE APPROPRIATE LANGUAGE OF PLACE VALUE IS USED, UNDERSTOOD AND THE VALUE OF EACH DIGIT IS EXPRESSED.* It is not 2 + 4, it is 20 + 40 is 60 or 2 tens + 4 tens is 6 tens, which is 60*

6. COLUMNAR ADDITION ('carrying')

Year 3 onwards

427 + 254 = 681

```
  4 2 7
+ 2 5 4
-------
  6 8 1
    1
```

Working from right to left:
"7 + 4 is 11. Partition the 11 into 10 and 1, 'carry' the ten into the tens column, writing it as 1, below the line, to represent one ten." *It is NOT "carry the 1"

"1 ten added to 2 tens and 5 tens is 8 tens" or "10 + 20 is 30; plus 50 is 80". Write this as 8 in the tens column to represent 8 tens or 80.

"4 hundred + 2 hundred is 600". Write this as 6 in the hundreds column to represent 600.

*Digits must be expressed as their appropriate values, NOT as single-digits i.e. *20* or *2 tens* NOT '2'.

PROGRESSIVE STEPS FOR THE COLUMNAR ADDITION:

- Not crossing tens — e.g. 432 + 246
- Crossing tens — e.g. 527 + 354
- Crossing hundreds only — e.g. 378 + 431
- Crossing tens and hundreds — e.g. 875 + 247

Year 3: 3-digit numbers
Year 4: 4-digit numbers
Years 5 & 6: 5-digit numbers, including decimals

IT IS IMPORTANT TO PROGRESS THROUGH EACH STEP FOR EACH STAGE OF CALCULATING ADDITION, REVERTING BACK TO THE FIRST STEP EACH TIME A NEW STAGE BEGINS.

LIKEWISE, WHEN EXTENDING TO ADDITION OF DECIMALS, REVERT BACK THROUGH THE STAGES OF PROGRESSION FROM 'CHAIN' TO ADDING MOST THEN LEAST SIGNIFICANT DIGITS BEFORE THE STANDARD WRITTEN METHOD.

USE THE FOLLOWING STEPS THROUGH EACH OF THOSE STAGES OF PROGRESSION FOR ADDING DECIMALS (Year 5+):

Jottings show partitioning & recombining ('chain') stage of progression.
Number/progressive steps are the same regardless of the stage of progression.

Years 5 & 6

One decimal place (1 d.p.)

- Not crossing ones (units) e.g. 1.3 + 1.4

- Crossing ones e.g. 3.5 + 1.7 = 5.2

$$3.5 + 1.7 = 4 + \frac{12}{10} = 4 + 1.2 = 5.2$$

When adding decimal fractions, express as fractions, i.e. Say 'tenths' not 'point …', i.e. Say "3 tenths add 4 tenths is 7 tenths" not "point 3 + point 4 = point 7." Expressing the fractional size is more meaningful.
When crossing ones, express this as:
"5 tenths add 7 tenths equals 12 tenths, which is 1 whole one and 2 tenths, which is 1.2"
Remind pupils that it is not 0.12 because it that is less than a whole one.

Note: We do not say "zero point twelve". It is "zero point one two".

Two decimal places (2 d.p.)

- Not crossing ones (units) or tenths e.g. 1.14 + 5.35 (as for 1d.p. above)

- Crossing tenths only e.g. 1.28 + 2.34 = 3.62

$$1.28 + 2.34 = 3 + \frac{62}{100} = 3.62$$

The assumption is that, if adding decimals, the pupils are able to add pairs of 2-digit numbers mentally. Also, that the value of hundredths in a decimal fraction with 2 decimal places is FULLY understood. Use this knowledge to add hundredths:

Say, " 28 hundredths add 34 hundredths is 62 hundredths, which is 0.62".

- Crossing ones only, e.g. 2.42 + 1.84 = 4.26

$$2.42 + 1.84 = 3 + \frac{126}{100} = 3 + 1 + \frac{26}{100} = 4.26$$

Years 5 & 6

Remember: ALWAYS PRESENT CALCULATIONS HORIZONTALLY IN ORDER TO CONSIDER MENTAL CALCULATIONS FIRST.

- Crossing ones and tenths e.g. 1.75 + 4.47 = 6.22

$$1.75 + 4.47 = 5 + \frac{122}{100} = 5 + 1 + \frac{22}{100} = 6.22$$

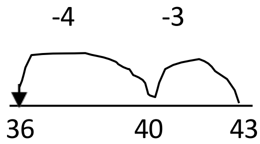
SUBTRACTION

It is important that children understand subtraction as 'take away' <u>and</u> as 'finding the difference' and to be able to interpret the context when faced with a subtraction calculation or problem. Even with 'finding the difference', a number may still be taken away and then we count on to find how much is left (the difference).

1. **COUNTING <u>BACK</u> IN 1s**

2. **COUNTING <u>BACK</u> (for a large difference** i.e when subtracting a small number)

43 – 7 = 36



Using <u>known facts and place value,</u> partition 7 into 3 + 4 to land on the multiple of 10 and continue counting back.

Year 1

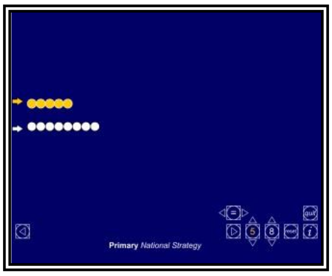
When dealing with larger numbers (beyond subtraction of 'teens' numbers, where place value and known facts can be used quickly) **then <u>COUNTING ON TO FIND THE DIFFERENCE</u> is then used as the most RELIABLE and EFFICIENT strategy.**

1. **COUNTING <u>ON</u> IN 1s**

Year 1

2. **COUNTING <u>ON</u> (initially for a small difference** i.e. 2 numbers which are close together, subtracting a large number) **COUNTING ON TO FIND THE DIFFERENCE**

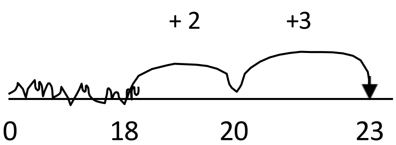
NB: It is important to spend a lot of time (initially in Year 1) on the <u>concept of 'difference'</u>. This can be demonstrated in using <u>concrete resources</u> by first comparing children's features (hair colour, glasses, cardigan or jumper, etc.) then moving on to ordering children's heights and then towers of cubes (in single colours) and other counting equipment.
Use the language *"How much more…?"*



<u>Moving onto</u> a middle step between the concrete and the pictorial: use the <u>Difference ITP</u> to demonstrate 'finding the difference'.

After lots of concrete practice and pictorial recording, <u>moving onto</u> the number line jotting:

23 – 18 = 5



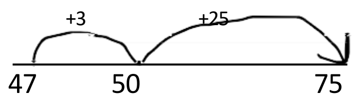
<u>Initially</u>, model the 'taking away' of the smallest number by 'scribbling out' the numbers from zero to that number to demonstrate that we are finding the difference; *how much more…?*

ALWAYS use 'landmarks' that are multiples of 10, i.e. <u>land on the next multiple of 10</u> (or 100 when doing calculations with larger numbers) using known facts to work out the size of the jump. <u>Write the landmarks on first</u> then do each jump, labelling the size above the jump each time (i.e. not at the end).

It is vital that children have an idea of the position of numbers in the number system to be able to recognise when to count on, when numbers are close together or near a multiple of 10, 100 or 1000. The <u>hundred square is NOT a suitable model</u> for this. The bead bar, which is a linear model, is far clearer.

<u>Year 2 TAF:</u> Subtract any 2 two-digit numbers using an *efficient strategy*, explaining their method verbally, in pictures or using apparatus (e.g. 72 – 17). Two jumps (to the next ten and on to the largest number) would be most *efficient.*

75 – 47 = 28

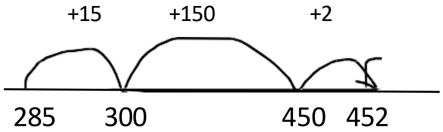


Years 2 & 3 and mentally through KS2 for subtraction of 2-digit numbers

COUNTING UP ON A NUMBER LINE SHOULD BE USED AND EXTENDED THROUGHOUT THE SCHOOL AS A MENTAL STRATEGY FOR SUBTRACTION (with a number line support if needed).

- larger numbers

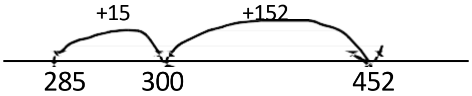
452 – 285 = 167



Again, ALWAYS use ‘landmarks’ that are multiples of 10, i.e. land on the next multiple of 10 (or 100 when doing calculations with larger numbers) using known facts to work out the size of the jump. Write the landmarks on first then do each jump, labelling the size above the jump each time (i.e. not at the end).

- larger numbers with more efficient recording

452 – 285 = 167

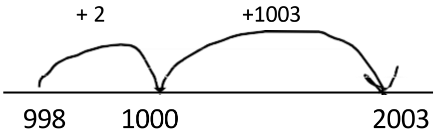


Initially with jottings (number line) then as a mental strategy through KS2 where possible

The expectation is that this will eventually be carried out MENTALLY in just two steps (‘jumps’).

- larger numbers but where the numbers are close to multiple of 100 or 1000. This should then be carried out MENTALLY.

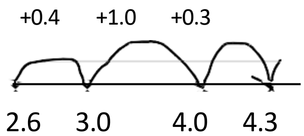
2003 – 998 = 1005



Remember: ALWAYS PRESENT CALCULATIONS HORIZONTALLY IN ORDER TO CONSIDER MENTAL CALCULATIONS FIRST.

- decimals

4.3 – 2.6 = 1.7

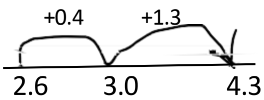


Revert back to this stage (counting up on a number line) when using decimals, even if at a higher stage of progression for subtraction of whole numbers, as it helps clarify the size of decimals and gives a clear visual image to emphasise the place value involved.

Years 5 & 6

Extend to carrying out with a minimum of 2 jumps’ (more efficient recording)

4.3 – 2.6 = 1.7



If the calculation should be possible mentally then do not give it to practise vertical calculation, e.g. TU - TU should not be calculated vertically. Consider use of numbers carefully, including the progressive steps within a stage. When secure, teach children to choose the most appropriate method and include calculations that can be carried out mentally/with a jotting so that children ‘spot’ them.

NB: THE FOLLOWING METHOD HAS NO PROGRESSIVE LINK FROM ANY PREVIOUS METHOD OF SUBTRACTION BUT STANDS ALONE AS A SEPARATE METHOD:

3. SUBTRACTION BY EXPANDED DECOMPOSITION (from the end of Year 3 and only if clear understanding of place value and correct vocabulary usage during the process of calculation; as well as an excellent recall of required addition and subtraction facts for Year 3+)

Year 3

Remember: ALWAYS PRESENT CALCULATIONS HORIZONTALLY IN ORDER TO CONSIDER MENTAL CALCULATIONS FIRST.

- Subtracting with no repartitioning needed:

345 – 123 = 222

```
   300  +  40  +  5
 - (100 +  20  +  3)
 ___________________
   200  +  20  +  2
```

Partitioning each number and working from right to left, subtracting the bottom number from the top. Express each part as its value represented, i.e ."40 – 20".

- Moving onto subtracting with repartitioning of tens only:

Practise **repartitioning** a number before teaching this stage of progression, e.g. lesson starters, partitioning in different ways, e.g. 57 = 50 + 7; 57 = 40 + 17

252 – 114 = 138

```
   200  +  50  +  2                   40       12
 - (100 +  10  +  4)        →   200 + 5̶0̶  +  2̶
 ___________________          - (100 + 10  +  4)
                   ?          _____________________
                                100  + 30  +  8
```

Year 3

Again, partitioning each number and working from right to left, subtracting the bottom number from the top. Where the subtraction is not possible i.e. 2 – 4 can't be done (without giving a negative number) the next value is "REPARTITIONED". So, "repartition 50 + 2 into 40 + 12". It is important to cross out the whole number and replace completely. Do NOT put a 'one in the air'! (It is not a 1, it is a 10.) Then repeat the subtraction process, this time "12 – 4 = 8" and "40 – 10 =30"

PROGRESSIVE STEPS FOR EACH STAGE OF EXPANDED DECOMPOSITION:

- Not repartitioning tens e.g. 458 - 436
- Repartitioning tens e.g. 547 - 328
- Repartitioning hundreds only e.g. 635 - 143
- Repartitioning tens and hundreds e.g. 725 - 437

Remember, always present calculations horizontally and consider mental/jottings before using a vertical written method. Do not give pupils calculations they can do with a mental strategy if you want them to practise a vertical method. Encourage pupils to look out for alternative strategies and allow them to use these.

4. SUBTRACTION BY STANDARD DECOMPOSITION

546 – 328 = 218

```
   5 ³4̶ ¹6
 - 3  2  8
 _________
   2  1  8
```

It is still vital that the correct language of place value is used. The tens are REPARTITIONED (not "'borrow' a 1" and it is not "3 takeaway 1" but "300 takeaway/subtract/ minus 100."

By Year 4

- Extend to decimals: **Years 5 & 6**

2.52 – 1.14 = 1.38

```
  2.00   +  0.50   +  0.02
- (1.00  +  0.10   +  0.04)
                        ?
```

→

```
            0.40        0.12
  2.00   +  0.50   +  0.02
- (1.00  +  0.10   +  0.04)
  1.00   +  0.30   +  0.08
```

→

```
   2. 4 5 1 2
 - 1.   1   4
   1.   3   8
```

PROGRESSION FROM MENTAL TO WRITTEN METHODS OF MULTIPLICATION AND DIVISION

Fluency in the recall of key number facts is a crucial aspect of expected standard and underpins much of the mathematics curriculum. Without this fluency, children are hindered in moving on in many areas of the curriculum, which rely on a good grasp of number. They will not meet expected standard and will struggle to cope with basic assessments.

AS WITH ADDITION AND SUBTRACTION, BEFORE PROGRESSING THROUGH THE STAGES OF CALCULATION, THERE ARE KEY PREREQUISITES SKILLS AND STRATEGIES TO BUILD ON:

Learning

- It is crucial to know or be able to derive key number facts –

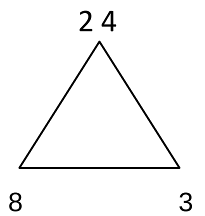
  Understand and use doubling and halving

  ×/÷ 10 (as moving a place to the left/right NOT "add a zero" etc!!)
- Place value and partitioning MUST be clearly understood and explained using the appropriate mathematical vocabulary.

Teaching

- The number line, the use of arrays and arrow jottings must be modelled as images to support calculation from Year 1 to Year 6.
- Jottings must be modelled as a clear image/strategy for mental calculation.
- Teach the 'three related numbers' so that links between the two operations are recognised,

  e.g.



8 x 3 = 24
3 x 8 = 24
24 ÷ 8 = 3
24 ÷ 3 = 8

Always present calculations horizontally in order to consider mental calculations first.

Always think:
1. *Can I do it mentally?*
2. *Can I do it with a jotting?*
3. *Do I need a written method (vertical layout)?*

If the calculation should be possible mentally then do not give it to practise vertical calculation, e.g. 23 x 15 should not be calculated vertically. Consider use of numbers carefully. When secure, teach children to choose the most appropriate method and include calculations that can be carried out mentally/with a jotting so that children 'spot' them.

MENTAL STRATEGIES

Learning times-tables, while making calculating more efficient, may not be possible for EVERY child as not ALL children will be able to learn ALL multiplication facts. However, strategies to calculate the facts not yet recalled ARE essential:

× 2	double	÷ 2	halve
× 4	double-double	÷ 4	half and half again
× 8	double-double-double	÷ 8	half, half and half again

Years 1 & 2 onwards

Year 3 onwards through KS2

Model jottings for halving and doubling and use known facts and place value:

½ of 6 4 = 3 2
½ (or h) ½ (or h)
30 + 2

“Half of 6 tens or half of 60 is 3 tens or 30”
“Half of 4 is 2.”

4 2 x 2 (or double 42) = 8 4
x2 (or d) x2 (or d)
80 + 4

“Double 4 tens is 8 tens or 80”
“Double 2 is 2.”

Year 2 onwards

Where the number of tens (or hundreds) is odd and the fact unknown, use known facts to derive the new fact:

e.g. ½ of 7 6 = 38
½ (or h)
(60 + 10)
½ ½
30 + 5 + 3

Year 3 onwards, including larger numbers

Where half of an odd number of tens is unknown, partition into an even number of tens plus 10, e.g. partition 70 into 60 + 10 and halve the two parts separately.

Stop using the arrows as soon as the concept is grasped. The arrows act as a reminder model and are not needed once the child is secure with the strategy. They may then just jot down ‘holding numbers’ – numbers they can’t hold in their head – to help them ‘see’ the answer.

Using best friend (10 and second-best friend (h/d), teach and learn that:

Year 3 onwards: × 5 ½ of × 10 (× 10 then halve it)

Years 5 & 6:

× 50 ½ of × 100 (× 100 then halve it)

× 25 ¼ of × 100 (× 100 then ½ and ½ again)

Introduce by modelling halving groups of a number using the **Multiarray ITP**.
Use a jotting at first (see below) then write down the first answer as a ‘holding number’ and calculate mentally.

× 12 × 10 plus × 2 (double)

× 15 × 10 plus ½ of × 10

Model that x 5 is half of x 10 is using the multiarray ITP. If you halve 10 groups of a number, you get 5 groups of that number,

e.g. 5 x 28 = 140

½

28 x 10 = 280

e.g. **Year 3 onwards** **Years 5 & 6**

3 2 x 5 = 160
↓ x10
320
↓ h
160

2 4 x 5 0 = 1200
↓ x100
2400
↓ h
1200

2 8 x 2 5 = 700
↓ x100
2800
↓ h
1400
↓ h
700

NB Strategies and jottings are the same for <u>dividing 2-digit numbers by 5, 50 and 25</u> but in reverse:

÷ 5 ÷ 10 and double the number

÷ 50 ÷ 100 and double the number

÷ 25 ÷ 100 and multiply by 4 (double, double)

Year 3 onwards

Years 5 & 6

<u>More mental strategies to be taught when multiplying 2-digit numbers, again using best friend (10 and second-best friend (h/d):</u>

× 12 × 10 plus × 2 (double)

× 15 × 10 plus ½ of × 10

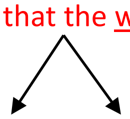
Years 5 & 6

e.g.

2 4 x 1 2 = 2 8 8
x 10 ↙ ↘ x 2 (or d)
240 + 48

3 2 x 1 5 = 4 8 0
x 10 ↙ ↘ x 5 (by halving the x 10 answer)
320 + 160

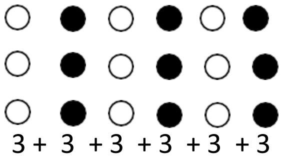
It is important that the arrows are <u>joined</u> in these jottings to show that the <u>whole</u> number is multiplied and not its partitioned parts.



For all of these arrow jottings. stop using the arrows as soon as the mental concept is grasped. The arrows act as a reminder model and are not needed once the child is secure with the strategy. They may then just jot down 'holding numbers' – numbers they can't hold in their head – to help them 'see' the answer.

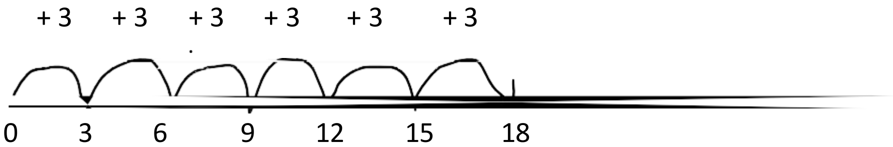
MULTIPLICATION

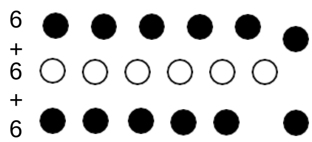
1. ARRAYS



shows 3 × 6 = 18

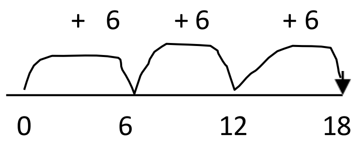
"3 multiplied by 5" is NOT "3 lots of 5" but "3 grouped 5 times". The 3 is the times-table, 5 is 'happening to it'; the 3 is being operated on, e.g. for "3 multiplied by . . .", we start with the 3 and repeat it 5 times.



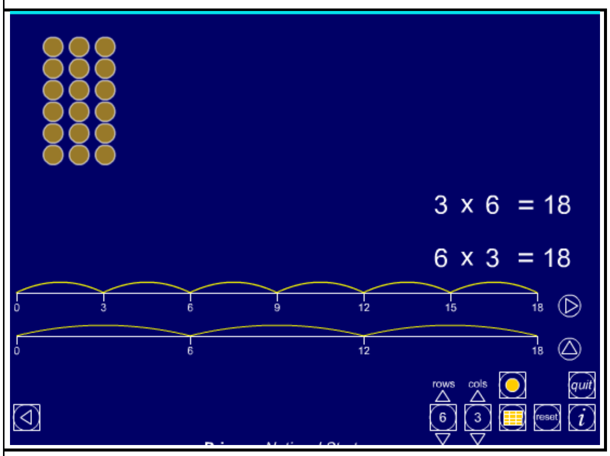


shows 6 × 3 = 18

"5 multiplied by 3" or "5 grouped 3 times". This is the 5 times table. The 5 is being operated on. It is *not* "5 lots of 3"



Years 2 & 3 for unknown times tables only



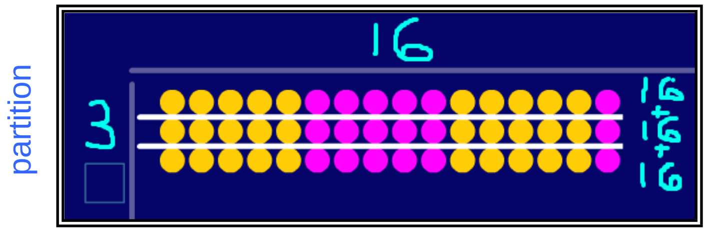
Use **Multiplication facts ITP** to model the link between the array and repeated addition on a number line.

Concrete representations are vital:
Use wrapping paper arrays/egg boxes and multilink cubes etc. "Show me 3 x 4, 6 x 3" etc. Draw the matching number line.

2. PARTITIONING

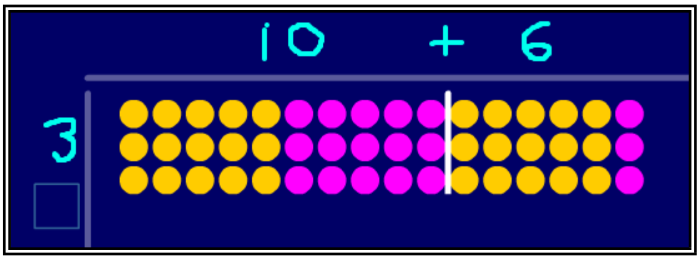
Use Multiarray ITP to model how 16 is multiplied three times: 3 rows (groups) of 16 = 16 + 16 + 16

16 x 3 is "16 grouped 3 times"



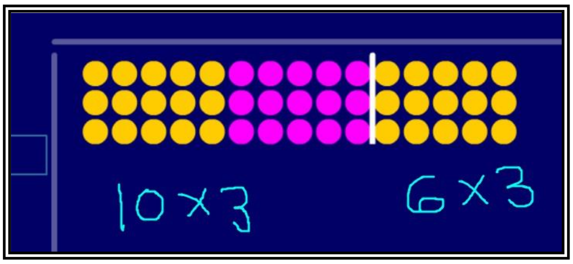
Also use **Multiarray ITP to model** how 16 is multiplied three times:
3 rows (groups) of 16 = 16 + 16 + 16

Also, use **Multiarray ITP to model** the 10 and the 6 both being multiplied by 3.

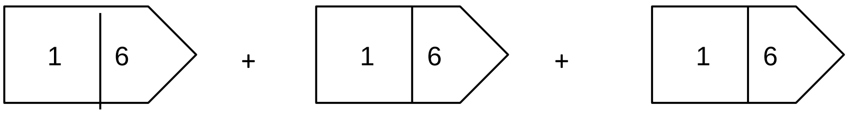


Year 3

so "10 grouped 3 times and 6 grouped 3 times"

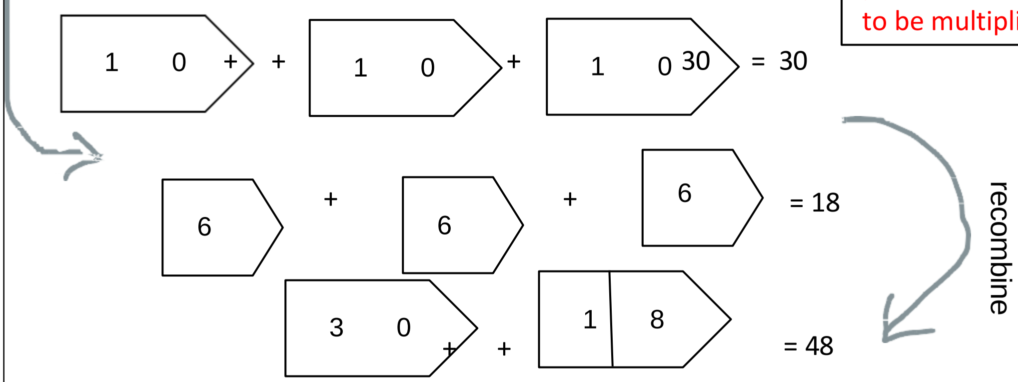


Partition 3 groups of 16:



So, partitioning out the 16 into 10 + 6 and multiplying both parts by 3.

Model and practise with **place value arrow cards and whiteboards** to show how each part of the number (partitioned part) needs to be multiplied by 3.



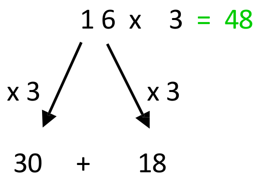
(using times-table facts or repeated addition)

Year 3

Pupils do not need to record like this as it is inefficient (see next point for how to record).

- Once partitioning concept understood, move onto recording as:

Year 3 onwards (mental strategy)



"10 multiplied by 3 is 30".

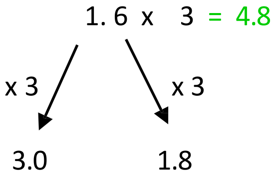
"6 multiplied by 3 is 18*

"30 + 18 = 48" (Only write this answer by the original calculation. Do not write it twice).

The arrows must point to the separate digits, the partitioned parts of the number: the tens and the ones.

This mental jotting can be used for all TU x U calculations.

This also works for decimals, e.g. 1.6 x 3



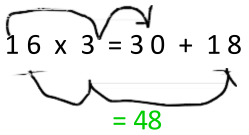
Years 5 & 6

0.6 x 3 is "6 tenths x 3 = 18 tenths
= 1.8" (one whole one and 8 tenths)

and/or using known facts that:
"0.6 x 3 is 10 times smaller than 6 x 3; 1.8 is 10 times smaller than 18."
This is NOT 'putting back' a decimal point!

If pupils do not have a clear understanding of tenths to be able to do this then then they are not ready for multiplication of decimals.
See addition of decimals for language/steps.

This becomes more efficient, in a similar way to adding two-digit numbers and can then be done without the arrows and, eventually, mentally, using known multiplication facts.



"10 x 3 is 30"
"6 x 3 is 18"

Year 3 onwards (mental strategy)

This mental jotting can be used for all TU x U calculations (from Year 3 and 4). **However, pupils also need to know HOW to carry out short multiplication.**

SHORT MULTIPLICATION is merely a formal way of writing down the above mental strategy (of partitioning to multiply) in a formal vertical presentation:

Expanded layout of short multiplication

16 x 3 = 48

```
  16
x  3
  18
  30
  48
```

"6 multiplied by 3 is 18* (or "3 multiplied by 6" now that the concept of commutativity is secure). Possibly, better to say "three sixes"

"10 multiplied by 3 is 30" (or "3 multiplied by 10"). Possibly, better to say "three tens"

"30 + 18 = 48"

Year 4: 2- or 3-digit x 1-digit numbers
Years 5 & 6: 4-digit x 1-digit numbers

Compact layout of short multiplication

16 x 3 = 48

```
  16
x  3
  48
  1
```

"3 sixes are 18"; place the 8 in the ones column, carry the ten.

"3 tens are 30; add the carried ten is 40." Place the 4 (tens) in the tens column.

Remember: ALWAYS PRESENT CALCULATIONS HORIZONTALLY IN ORDER TO CONSIDER MENTAL CALCULATIONS FIRST.

Extend to decimals (Year 6):

2.4 x 3 = 7. 2 Either mentally (as above with arrow jottings as initial support) or by mentally partitioning to give:

Year 6 written method if not possible mentally/with jotting

2.4 x 3 = 6 + 1.2
= 7.2

or
2.4
x 3
7.2
1

Put the decimal point in the equals bar in first.

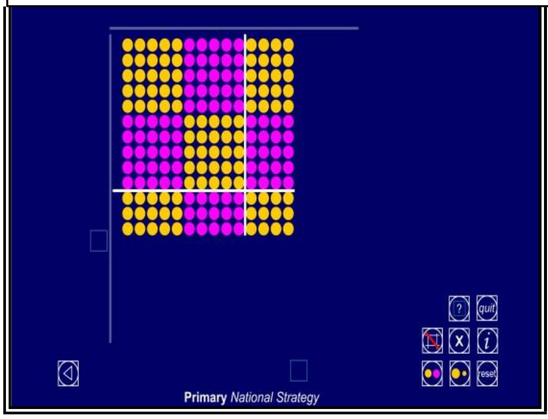
-

REMEMBER, short multiplication is a 'need-to-know' not a 'need-to-do' when other strategies (mental/jotting, e.g. arrows) are more efficient. Pupils must be encouraged to carry out the most appropriate, efficient AND reliable method for the numbers and for their abilities.

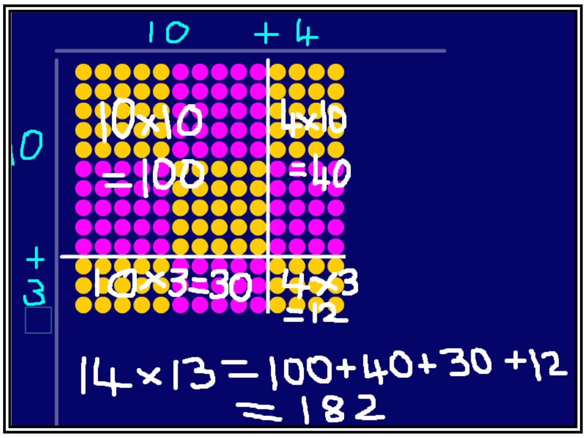
Year 5

3. MULTIPLICATION GRID beginning with 2-digit x 2-digit numbers (Year 5+)

BEFORE moving onto written recording in a grid, it is essential to model the concept of partitioning to find the area of each rectangle in order to find the area of the whole. This visually represents what happens when we multiply two numbers i.e. find the number in an array – find the area of a rectangle.



Use Multiarray ITP to model partitioning into tens and ones, using the familiar visual pattern of 5s. Calculate the 'area' of each rectangle by multiplying the number of rows by the number of columns.



The link to **finding the area of a rectangle** is vital here so that the concept of multiplication is truly understood and not carried out by rote.

- Moving onto:

REMEMBER, ALWAYS PRESENT CALCULATIONS HORIZONTALLY IN ORDER TO CONSIDER MENTAL CALCULATIONS FIRST.

Again, if the calculation should be possible mentally then do not give it to practise vertical calculation, e.g. 23 x 15 should not be calculated vertically. Consider use of numbers carefully. Avoid numbers which involve x 2, x 4, x 5, x 8.

1 6 x 1 7 = 272

	1 0	+ 6
1 0	10 x 10 = 100	10 x 6 = 60
+ 7	7 x 10 = 70	7 x 6 = 42

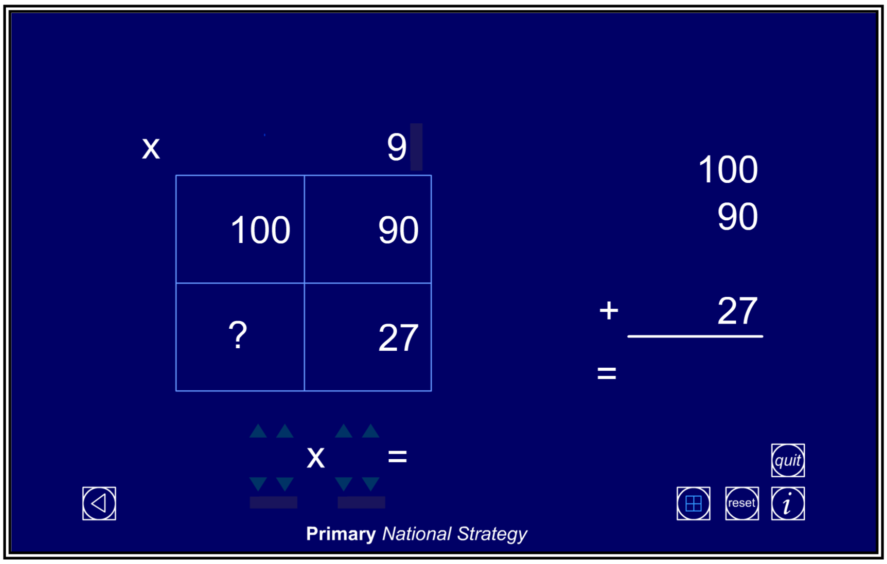
It is important to write the calculation in the grid for both the pupil and teacher to be able to identify errors made in multiplication facts or in the calculating the process.
It is also a reminder that the area of the rectangle is being calculated and the system is clear.

Try to adopt the system always starting each calculation with the number above (or to the side, as long as it is consistent) so that it is easy to check the correct calculations (and answers) have been carried out, e.g. ten and six x 10 and ten and

100 + 70 + 60 + 42

170 + 102 = 272

Where possible, use mental calculation strategies to calculate the total e.g. looking for known facts or adding the largest number first.



Year 5

Only use Multiplication grid ITP to assess understanding and application of the grid method by 'hiding' the question parts and 'revealing' some of the answer parts.

PROGRESSION:

1. TU x TU
2. HTU x U
3. HTU x TU
4. HTU x HTU
5. U.t x U

Revert back to this stage (the grid method) when using decimals, even if at a higher stage of progression for multiplication of whole numbers, as it helps clarify the size of decimals and gives a clear visual image to emphasise the place value involved.

Again, if the calculation should be possible mentally then do not give it to practise vertical calculation, e.g. TU x 15 should not be calculated vertically (see p.10 for mental jottings). Consider use of numbers carefully, including the differentiation steps within a stage. Avoid numbers which involve x 2, x 4, x 5 or x 8. When secure, teach pupils to choose the most appropriate method and include calculations that can be carried out mentally/with a jotting so that pupils 'spot' them.

Avoid incorrect use of mathematical language, e.g. correct the commonly-used "timesing" to "multiplying".

4. EXPANDED LONG MULTIPLICATION

1 6 X 1 7 = 272

```
     1 6
  x  1 7
     4 2     7 X  6
     7 0     7 X 10
     6 0    10 X  6
   1 0 0    10 X 10
   2 7 2
```

Initially, teach alongside the multiplication grid and show how each calculation links to each 'box' of the multiplication grid.

Tip: If TU x TU (i.e. 2-digit x 2-digit) then a '2 x 2' multiplication needs 4 calculations/ boxes (2 x 2 = 4). If HTU x TU (3-digit x 2-digit) then a '3 x 2' multiplication needs 6 calculations (3 x 2 = 6) etc.

Check that the correct calculations have been made by looking for the numbers involved (on the right) i.e. 7 x 16 (the first two) and 10 x 16 (the second two).

Year 5

Where possible, use mental calculation strategies to calculate the total e.g. looking for known facts or adding the largest number first. Otherwise, add as for column addition.

- Moving onto the formal compact method of **LONG MULTIPLICATION** **Years 5 & 6**

1 6 x 3 7 = 272

```
    1 6
 X  3 7
 1  1 2    x 7
    4
 4  8 0    x 30
 1
 ------
 5  9 2
```

Year 5: 3- then 4-digit x 2-digit numbers
Year 6: 4-digit x 2-digit numbers

"7 times 6 is 42; carry the 4 tens (put 4 below in the tens column) and put the 2 in the ones column.
7 times 1 ten is 7 tens (70); plus the 4 carried tens is 11 tens so 110 so 1 in the hundreds column and 1 in the tens column."

Then, as we are multiplying by multiples of tens next, we know there are no ones so already know there will be a zero in the ones column on the next row. After we have multiplied by 10 (placed the zero in the ones column to indicate no ones for a multiple of ten), cross out the zero in x 30 to remind that it has been done.
We do NOT just "put a zero." It is important that it is understood why there are no ones. We have multiplied by ten.

Having multiplied by 10, we can now multiply by (in this example) 3.

3 x 6 = 18 so carry the ten (ten tens, 100) in the hundreds column and put the 8 tens in the tens column. Then 3 x 10 is 30 (30 tens, 300) plus the carried one (hundred) is 4 (hundred)/

Add up the columns from right to left in line with column addition procedures.

Again, where possible, use mental calculation strategies to multiply, e.g. if x 17 then just multiply the whole number by 10 not each part for the second row calculation; for x 26, multiply by 20 by doubling and multiplying by 10 in one go rather than multiplying each partitioned part by 2 and 'moving up a place so there is a zero in the ones column).

Tip: The number with the fewest digits goes below the other number. The number of digits determines the number of rows of calculations, e.g. 35 x 452 so 35 goes below the 452 and two rows of calculations will be needed – one calculation for x 5 and the other for x 30.

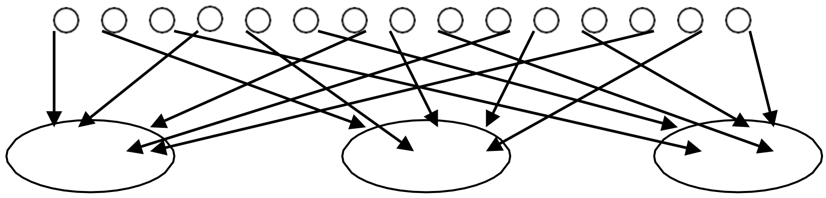
NB: Children need to write the carried digits in small writing on the line, NOT use a whole row for the carried digits as it appears above.

NB: IT IS NEVER NECESSARY TO DO A ROW TO MULTIPLY BY ZERO!

DIVISION

1. SHARING

15 ÷ 3



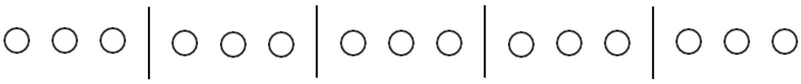
“15 shared between 3”

5 + 5 + 5 or 3 sets of 5
or 5 ‘each’

Year 2

2. GROUPING

15 ÷ 3

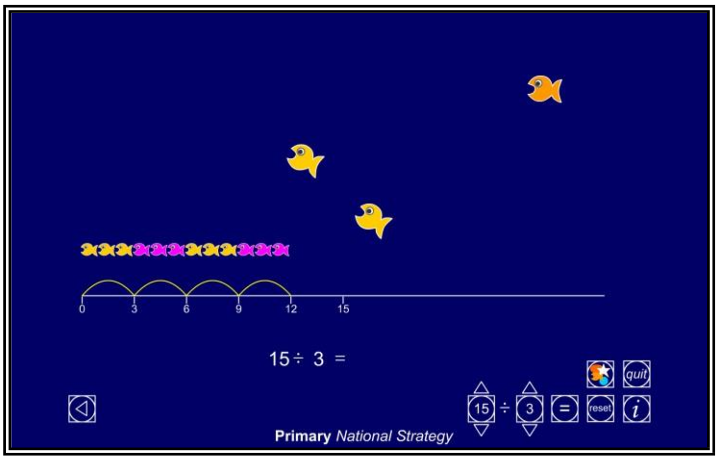


“15 grouped in 3s”

3 + 3 + 3 + 3 + 3 or 5 groups of 3

Grouping can be easily modelled on the 100-bead bar.

ONCE LARGER NUMBERS ARE USED, GROUPING IS MORE EFFICIENT AND RELIABLE THAN SHARING.



Repeated addition (easier to count on than count back).

Model using the Grouping ITP to make the link between grouping on the number line and repeated addition.

Grouping on a number line should NEVER be used for known times-tables facts as it would be a pointless exercise if the related division fact is known mentally. Nor should it be used if the child can count on in steps of the divisor, e.g. 3, 6, 9, as this will be quicker on their fingers than constructing a number line. Nor should it be used for dividing by 2, 4, 5 or 8 as there are mental strategies for these. Consider numbers and efficiency of calculation. Encourage decision-making so that children do not feel they HAVE to use a number line when they count up in the heads more quickly.

18 ÷ 3 = 6

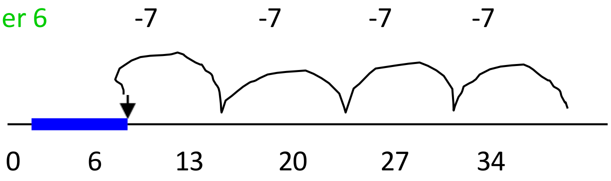
+3 +3 +3 +3 +3 +3

0 3 6 9 12 15 18

showing ‘6 groups of 3’

- Moving onto remainders:

34 ÷ 7 = 4 remainder 6



shown as '4 groups of 7 and 6 left over'

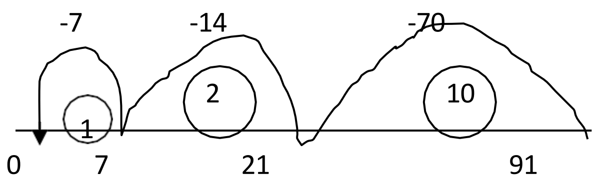
Repeated subtraction now shows the remainder more clearly than repeated addition as the remainder/left over is less than the divisor and no calculation is required to see how much is left over. It also helps avoid pupils making another group whatever the size of the reminder.

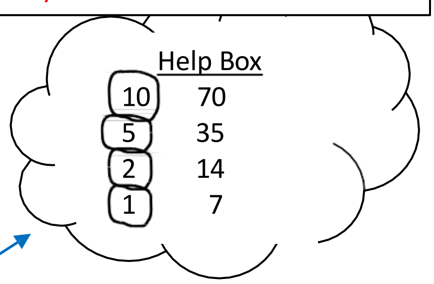
Year 3 for unknown times tables

- Moving onto division calculations involving larger numbers by **'CHUNKING' ON A NUMBER LINE** and using a **'Help Box' of key facts** based on best friend (10) and second-best friend (h/d).

Concrete representation: Multiples of the divisor are subtracted along the number line. This can be modelled by relating it to breaking of chunks (rows) of a chocolate bar, rather than cube-by-cube.

91 ÷ 7 = 13 (10 groups + 2 groups + 1 group)





The number of 'chunks' of the divisor being subtracted is written in a circle below the jump. The total of these numbers is the number of groups of the divisor in the larger number i.e. the answer.

- The Help Box ONLY needs x 10, x 5, x2 and x1 facts of the divisor. All other facts can be derived from these.
- Remember, x5 is calculated mentally by halving x10 and x2 is expressed as 'double'.
- By drawing the Help Box in a cloud (after the facts have been listed) it stays tidy by not being crooked lines of a box and does not slow down the process.
- The Help Box MUST be used before the calculation is carried out. If it is not needed then the calculation is probably too easy and the pupil could probably calculate by mentally chunking the number.

PROGRESSION:

1. TU ÷ U where the dividend is greater than the 12th multiple of the divisor so that it is not within known tables facts
2. HTU ÷ U

- **MENTAL CHUNKING**

192 ÷ 16 = 12

160 + 32

(10) (2)

Year 4 onwards

Partitioning 192 into the 10th multiple of the divisor and "some more" and recording the number of groups of the divisor in circles below.

- **MENTAL CHUNKING** leads to **SHORT DIVISION** when it is understood that it is multiples of ten (or hundred etc.) of the divisor and not single digits multiplied by the divisor.

657 ÷ 3 = 239

```
  2 1 9
3)6 5 ²7
```

512 ÷ 16 = 32

```
   0 3 2
16)5 1 ³2
```

1. "16 does not go into 500 a hundred times." Put a small 0 in the hundreds column to ensure the place is not used.
2. "16 goes into 51 (tens) three (tens) times, which is 48 (tens)" .. knowing three 16s are 48. Put a 3 in the tens column above the line to show this.
3. 51 tens – 48 tens = 3 tens remaining – show as a small, higher 3 in the tens column below the line.
4. "16 goes into 32 twice." Put a 2 in the ones column above the line.

Years 5 & 6: 3- then 4-digit ÷ 1-digit

- Moving onto **'VERTICAL CHUNKING'** for larger numbers e.g. HTU ÷ TU

192 ÷ 24 = 8

```
24)1 9 2
  -1 2 0     (5)
     7 2
  -  4 8     (2)
     2 4
  -  2 4     (1)
       0
```

(10)	240
(5)	120
(2)	48
(1)	24

Year 6:
4-digit ÷ 2-digit numbers

Subtraction calculations involved when removing each chunk of the divisor <u>SHOULD NOT</u> be carried out vertically – this is when most children make errors with division. Instead, calculate mentally (by counting up) or by using a jotting to count up on a number line.

The Help Box must still be used with the same key facts, based around 10, 5, 2 and 1. Other related facts may be calculated mentally from these e.g. x20 is double x10 but need not be written in the Help Box. When times-table/division facts are spotted (once the method is clearly understood and used with the appropriate use of mathematical language in explaining the process) then these maybe used to make the process more efficient.

The end of the SHORT OR LONG DIVISION process is when the calculation ends in zero or a number less than the divisor. If there is a remainder, this should be expressed as a fraction,

e.g. 292 ÷ 13 = 22 remainder 6
= 22 6/13

If the fraction's decimal equivalent is known then the decimal is used,

e.g. 203 ÷ 14 = 14 7/14
= 14.5

TIP: WHEN TO USE SHORT DIVISION AND WHEN TO USE LONG DIVISION – DECISION MAKING:

If the <u>2-digit divisor is less than the first two digits of the dividend</u> then use <u>short division</u>.

If the divisor is greater than the first two digits of the dividend then use <u>long multiplication</u>.

THE TERM 'BUS STOP METHOD' SHOULD NOT BE USED! It is vital that children know the difference between *short division* and *long division* and when to choose which method, depending on the size of the numbers. A generic 'nickname' of 'bus stop method' does not indicate which type of division is being used.

- <u>VERTICAL CHUNKING leads to **LONG DIVISION**</u>

Long division relies on the ability to multiply 2-digit numbers (when the divisor is a 2-digit number) which can be difficult. Using best friend (10) and second-best friend (h/d) facts helps to estimate and get closer to the answer without endless 'trial and improvement' long multiplication attempts to work out how many times the divisor 'goes into' the number.

Year 6:
4-digit ÷ 2-digit numbers

Each time, aiming to subtract the largest possible multiple of the divisor.

1440 ÷ 32 = 45

```
      0 0 4 5
32 | 1 4 4 0
   - 1 2 8
       1 6 0
     - 1 6 0
           0
```

At the side of the page, write the number of possible groups (in circles):
Always calculate <u>10x, 5x, 2x and 1x</u> the divisor before starting.

(10)	320	
(5)	160	(using ½ of 32 x 10)
(2)	64	(double the divisor)
(1)	32	

If these numbers are not large/small enough to subtract then use d/h to calculate other multiples of the divisor and using what has already been calculated, e.g. double 2x is 4x:

(4) 128 (using double 64)

If 8x was needed, we could use double 4x. If 6x was needed, we could use 5x plus 1x the divisor and so on.

If the <u>numbers at the top</u> are not put there until the end, the workings/estimations at the side mean that the multiples are not forgotten/lost.

<u>PROGRESSION:</u>
1. HTU ÷ TU
2. ThHTU ÷ TU

THE TERM 'BUS STOP METHOD' SHOULD NOT BE USED! It is vital that children know the difference between *short division* and *long division* and when to choose which method, depending on the size of the numbers. A generic 'nickname' of 'bus stop method' does not indicate which type of division is being used.

<u>REMINDER</u> TIP: WHEN TO USE SHORT DIVISION AND WHEN TO USE LONG DIVISION – DECISION MAKING:

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If the divisor is greater than the first two digits of the dividend then use <u>long multiplication</u>.